A Study of Heat and Mass Transfer Performance of Nanofluids in Various Geometries

Research Protocol

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Content

1. Title
2. Introduction
3. Literature Review
4. Justification for Selecting Research Area
5. Research Objective
6. Research Methodology
7. Expected Outcome
8. Work Plan
9. References

Research Protocol

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1. Title

A Study of Heat and Mass Transfer Performance of Nanofluids in Various Geometries

2. Introduction

Fluids have played an important role in this world, from plasma to the human body, car breaks, and heavy engineering work. Fluids can be in liquid form or gas form. Fluids have their natural characteristics known as fluid properties, which are viscosity, density, thermal conductivity, specific heat, etc. (Pati 2012):

Newton's law of viscosity: The shear stress is directly related to shear deformation.

$$\tau \propto \frac{du}{dy}$$

Or

$$\tau = \mu \frac{du}{dy}$$

Here μ represents dynamic viscosity, τ is shear stress, and $\frac{du}{dy}$ is shear deformation.

Based on Newton's law, we can classify fluids as Newtonian and Non-Newtonian fluids.

Newtonian and Non-Newtonian fluid

Newtonian fluids that obey Newton law of viscosity like water and air. If there is a non-linear relation between stress and strain, then it is called Non-Newtonian fluid such as butter, blood, toothpaste, etc.

Some types of Non-Newtonian fluids (Lavrov 2023; Deshpande et al. 2010).

(1) **Ostwald-de-Waele power-law fluid:** Initially Ostwald and De Waele proposed this model and Reiner described this model. The equation for this model

$$\tau = K\dot{\gamma}^n$$

K and n representthe consistency index and power-law index respectively. It is called Newtonian fluid ($n = 1$), Pseudo plastic fluid ($n < 1$), and Dilatant fluid ($n > 1$).

(2) Maxwell Fluid Model: Maxwell fluid model is a linear model that relates the elasticity of solids with the viscosity of nanofluids. The equation that describes the relation

$$\tau = -\alpha_1 \frac{\partial \tau}{\partial t} + \mu_0 \gamma$$

Where, τ stress tensor, γ rate of strain, μ_0 shear viscosity and α_1 is relaxation time when $\alpha_1 = 0$ the fluid becomes Newtonian fluid.

(3) Jeffrey Fluid Model: This model is an extension of Maxwell's fluid model with the derivative of rate of strain. The equation of this model

$$\tau = -\alpha_1 \frac{\partial \tau}{\partial t} + \mu_0 \left(\gamma + \alpha_2 \frac{\partial \gamma}{\partial t}\right)$$

Here α_2 is retardation time, when $\alpha_2 = 0$ then it converts into Maxwell's Model and when $\alpha_1 = \alpha_2 = 0$,fluid is a Newtonian fluid.

(4) Eyring Fluid Model: This model is used for those nanofluid that behaves like Newtonian fluids at low shear rates and when the shear rate is high it shows Non-Newtonian behavior. The equation of the model is

$$\frac{\tau_{xy}}{A} = -\sinh^{-1}\left(\frac{1}{B}\frac{dv_y}{dx}\right)$$

Here A, B are parameters.

(5) Casson Fluid Model: This model was developed by Casson to predict the flow behavior of pigment oil suspension. In this model viscosity is given by the square-root relation and the equation of the model is

$$v = \left(\sqrt{\tau_0/\dot{\gamma}} + \sqrt{m}\right)^2, v_{min} \leq v \leq v_{max}$$

In the engineering and industrial sectors the most popular thing which makes the role of fluid important in these sectors is transferring the heat. Initially, they use natural conventional fluids like engine oil,water, ethylene glycol, etc. which have low thermal conductivity.

To enhance their thermal conductivity we take a mixture of fluid with millimeter-micrometer-sized solid particles. But using these solid-liquid mixtures they face many problems, due to the large size diameter of solid particles, they settle down, or sometimes a cluster of the particles clogs the channel, and for higher concentration fluid we have to push hard for fluid flow.

Nanofluid and Hybrid Nanofluid

Nanofluid was invented by Choi in 1995 to enrich the thermal conductivity of mixtures, which are made from the suspension of nanosized particles into base fluids Nanosized particles can be metal (gold, silver, copper, etc.), metal oxide (Alumina, Titania, Silica, etc.), Carbon nanotubes (CNT), etc. If we use more than one type of nanoparticles in suspension then it is known as a hybrid nanofluid. These hybrid nanofluids exhibit higher thermal conductance than nanofluids or base fluids. In various industries, such type of fluid is used in various applications such as in the automotive industry as a coolant, solar water heating, nuclear reactor cooling, etc.

From previous research, it has been shown that the existence of these particles (nanosized) affects the thermophysical properties and flow behavior of nanofluids. Several different flow geometries have been used in the study of nanofluid including: (Schlichting et al. 2016)

(1) **Laminar flow**: For this flow generally, the Reynolds number is not more than its critical value i.e. 2300 e.g., Pipe flow, Flow over a plane surface.

(2) **Turbulent flow**: For this flow generally, the Reynolds number is not less than 4000 e.g., Flow in a channel with an obstacle, Flow over a rough surface.

Heat and Mass Transfer

Heat transfer is the interchange of thermal energy in systems with different temperatures between two or more objects. The three main ways that heat is transported are given below (Munson et al. 2006):

(1) **Conduction**: Conduction is the process through which heat transfers through a solid material or between two solid materials that are in touch with one another.

(2) **Convection**: When heat moves from a surface to a moving fluid. The movement of the fluid causes heat transmission. There are three major types of convection.

   (a) Natural convection: The flow of fluid due to variations in fluid density put on by buoyant forces brought on by temperature fluctuations called natural convection.

   (b) Forced convection: Forced convection occurs when the flow of fluid through a surface due to external force, causing heat absorption and subsequent cooling.

   (c) Mixed convection: Mixed convection is the simultaneous transport of heat in a fluid by forced and natural convection.

(3) **Radiation**: By use of electromagnetic waves, heat is transferred by radiation. Radiation, unlike conduction and convection, may occur in a vacuum with no medium.

Diffusion or advection techniques can be used to transport nanosized particles in a nanofluid, a process known as mass transfer. The mass transfer rate is affected by the nanoparticle's volumetric concentration, fluid velocity, and shape of the used system. Increasing both the volume of nanoparticles and the flow velocity of nanofluid can accelerate the mass transfer rate. Furthermore, the geometry of the structure is important in mass transfer. Using various types of nanofluids, we use various nanomaterials, which may affect the thermophysical properties.

(1) **Thermal conductivity**: The rate of heat transport per unit area when the gradient of the temperature is normal to the cross-section area.Thermal conductivity increases by adding nanoparticles in base fluid (or increasing concentration) due to Brownian motion and the interfacial layer (nanolayer).

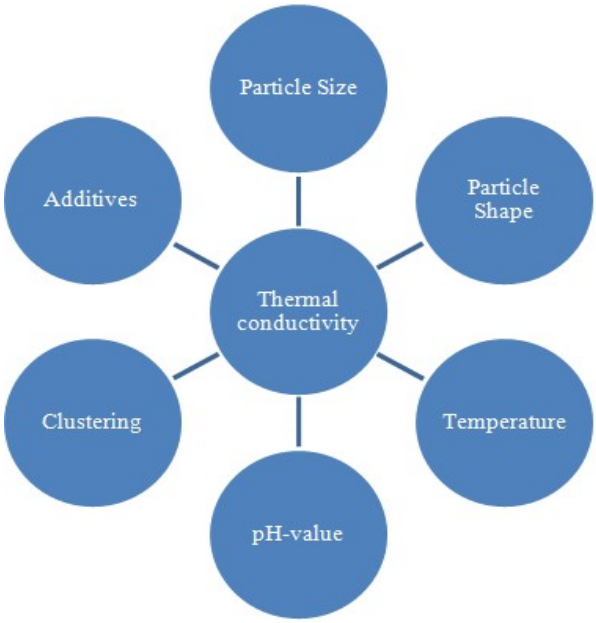

Fig. 1: Factors affecting Thermal Conductivity

(2) **Viscosity**: Viscosity is another thermophysical characteristic of nanofluids in heat transfer applications. Various factors influence viscosity, including volume concentration, temperature, and particle size. (Fig.2)

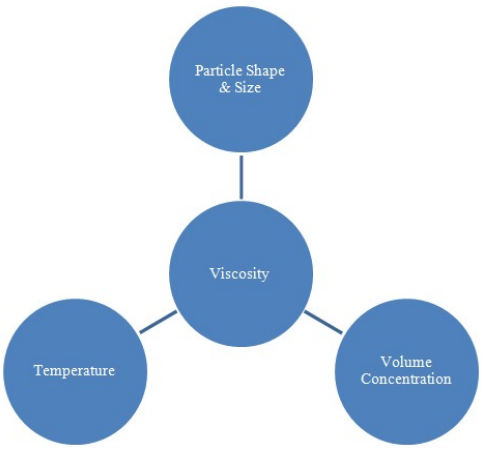

Fig. 2: Factors affecting Viscosity

Heat and Mass transfer models

(1) Bungirano model: Bungirano (2006) was used to explore the convective heat transfer phenomena caused by Brownian motion and thermophoresis of nanoscaled particles in nanofluids. The random movement of particles in a fluid, known as Brownian motion, affects the behavior of nanofluids, and thermophoresis is the movement of particles in reaction to temperature gradients.

(2) Tiwari and Das model: By taking into account nanoscale particle size, volume fraction, and particle-fluid interaction in an empirical equation, the Tiwari-Das model (2007) predicts nanofluid thermal conductivity. The equation of this model is

$$K_{eff} = K_{base}(1 + \varphi\beta)$$

K_{eff}, K_{base}, φ, β are thermal conductivity of the nanofluid and base fluid, volume fraction and parameter that depends on shape and size respectively

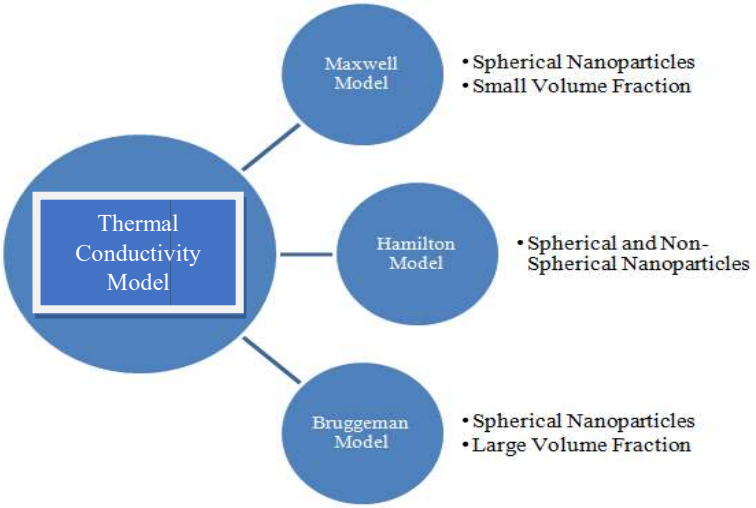

Fig. 3: Fundamental models for thermal conductivity

Principle Equations

There are fundamental differential equations that are always solved while studying fluids. (Cunet sert, 2023)

1. Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

It is also called an equation of conservation of mass.

2. Energy Equation:

$$\rho \left(\frac{\partial h}{\partial t} + \nabla \cdot (h \vec{v}) \right) = -\frac{Dp}{Dt} + \nabla \cdot (k \nabla T) + \epsilon$$

Where h, T, and ϵ are specific enthalpy, absolute temperature, and dissipation function respectively and the above is energy conservation equation.

3. Navier-Stokes Equation: Also known as the conservation of linear momentum equation.

$$\rho \left(\frac{D \vec{v}}{Dt} \right) = -\nabla p + \nabla \cdot \vec{\tau} + \rho \vec{f}$$

Here $\vec{\tau}$ is viscous stress and $\vec{f}$ is force per unit mass.

3. Literature Review

Many researchers and scientists have done experimental and numerical findings on the flow of nanofluids with different geometries. Many researchers tried to find a good and stable enhancement in the rate of heat transference. They take a huge number of environmental or natural factors under consideration like shape, size, the nanoparticles concentration, pH value of base fluid, the temperature of nanoliquid during the measurement, clustering of nanoparticles, and a period between synthesis of nanofluid and experiment. Teng et al. (2010) and Paul et al. (2010) reported that thermal conductance and particle size are inversely related. Murshedet al. (2005) said that spherical nanoparticles have lesser thermal conductivity than that of rod-shaped nanoparticles. Yu et al. (2009) and Duangthongsuk (2010) noticed that thermal conductivity is positively related to temperature. Initially, to know the impact of the acidity of pure liquid on thermal conductivity, Xie et al. (2002) considered nanofluids using Aluminum-water nanofluid and found that an increase in acidity of pure fluid decreases the thermal conductivity. Keblinski et al. (2002) and Hong et al. (2006) explored the influence of clustering; due to the clustering of nanoscaled particle thermal conductivity diminishes. Eastman et al. (2001) observed thermal

conductivity in two cases without additives and took additives into the fluid and thermal conductivity enhanced with additives.

Viscosity is another important characteristic of fluids. Much research gives us evidence about viscosity and how it can be concerned by the size, shape, & surface area of nanosized particles along with temperature and volume concentration. Murshed et al. (2008) and Sundar et al. (2013) studied the viscosity enhancement with nanoparticle concentration, which is higher than the base fluid. Nguyen et al. (2008) did experimental work with alumina-water nanofluid and took two sizes of nanoparticle and concluded that larger particles of alumina have a higher viscosity than the smaller particles of alumina nanofluid also assures the decrease in viscosity is due to temperature increase.

Various types of geometries are studied by researchers including flat tubes, circular tubes, triangular, rectangular ducts, vertical plates, etc. However, many of the researchers found that the shape and size of the geometry affect the heat conductivity of nanofluids significantly. Fiebig et al. (1994) determined from their research that the flat tube thermal exchanger has better efficiency than other circular tubes in transferring heat. Now numerical studies of heat transfer by Nusselt number (*Nu*) is a good approach to compare circular tubes, flat tubes, or other geometries.

Nield and Kuznetsov (2009) conducted a study on steady natural convection motion in a fluid-saturated porous medium with a uniform temperature gradient; they provided linear and quadratic regression formulas for the Nu at a fixed Lewis number (*Le*) of 10, with maximum errors of 12% and 1.4%, respectively. Kuznetsov and Nield (2013) have done this study by proposing an updated model that incorporates the effect of nanoparticle migration, for a more accurate prediction of the heat transfer coefficient. For *Le* is 10, The authors calculated linear and quadratic regression formulas for *Nu* with maximum errors of 1% and 12%. Kuznetsov and Nield (2010) examined the nanofluid's motion on a vertical plate for this they used the Bungirano model (Bungirano 2006) and developed a linear and quadratic regression correlation for *Nu* at stable Prandtl number (*Pr*) at 10 and *Le* at 10 with maximum errors of 8% and 0.9%. They concluded that the *Nu* is decreasing with the Buoyancy ratio number (*Nr*), Brownian parameter (*Nb*), and Thermophoresis parameter (*Nt*). Kuznetsov and Nield (2014) proposed a revised model for Kuznetsov and Nield (2010), in this model they consider the nanoparticles motion at the boundary. They got that the *Nu* is independent of the *Nb*, as well as it decreases with *Nt*.

Namburu et al. (2009) investigate numerically three kinds of nanoparticle copper oxide, aluminum, and silicon oxide using ethylene glycol–water base fluid through a circular tube, heat flux is constant, and they found all the thermophysical properties differ as the temperature varies as well as the concentration. They got that at a very high value of Reynolds number (*Re*) (approx. 20000), the *Nu* is 1.35 times that the pure fluid and heat transfer coefficient increases by 1.75 times. Vajjha et al. (2015) studied numerically a 3-D turbulent fluid's flow in a flat tube automotive heat radiator with aluminum oxide and copper oxide with ethylene glycol water. For thermal conductivity, they developed new formulas with an error of 2.8% for aluminum oxide and 7% for copper oxide, density, and specific heat estimated by Xuan and Roetzel (2000). The heat transport coefficient moves in the same direction as the *Re* moves. At fix (Re) 5500, heat transfer coefficient at 6% volume concentration 61% (alumina) and 92.5% (copper oxide). The Nusselt number is proportional to 0.8 exponents of Reynolds number (for 3000-8000).

Khan and Hamid (2018) studied the Non-Newtonian Williamson nanofluid passes through a wedge–shaped geometry. The Bungirano model (Bungirano 2006) was employed to study the influence of Brownian motion and thermophoresis parameters on the motion of fluid, which is time-dependent. The researchers examined that the occurrence of nanoparticles in Williamson's nanofluids improves heat transfer performance and the fluid temperature decreases, while the mass transfer rate rises with nanoparticle concentration. The study highlighted that the wedge angle, *Pr*, and *Re* strongly affect the fluid flow, heat, and mass properties of nanofluids.

Khan et al. (2019), Lahlou et al. (2020), Chu et al. (2020) and Batool et al. (2022) investigated heat transport properties with radiation in different geometries and they all found that due to radiation, heat transfer increases because it provides a new dimension to transfer heat directly, from system to surrounding medium.

Iqbal et al. (2023) investigate the interaction between heat transfer and human impact of Powell-Eyring nanofluid in a curved channel using magnetohydrodynamics (MHD) in a radius-dependent magnetic field. Modified Darcy's law, Brownian motion, thermophoresis, thermal radiation, motile gyrotactic microorganisms, Ohmic heating, and variable characteristics are all included in the study. Model accuracy is improved by taking thermal slip conditions and velocity into account. Additionally, motile microbes increase heat transfer and stability of nanoparticles while adding complexity by affecting the overall dynamics of transport.

Reddy et al. (2023) investigated the different effects of MHD heat and mass transfer in a hybrid nanoliquid flowing across a stretchable/shrinkable surface. The nanoparticles of titanium oxide

and alumina in water are examined. In both unstable and stable conditions, temperature profiles are raised by higher nanoparticle volume fractions. Reddy and Sreedevi (2023) examined the impact of magnetic fields and thermal radiation on the transmission of heat in a water-carbon nanotube nanofluid between expanded rotating disks with convective boundary conditions and heat generation/absorption. Nagaraja et al. (2023) investigated solar radiation and chemical reactions using modified ternary hybrid nanofluids that circulate in opposing and assisting flows over an exponentially stretched surface. Raghunath (2023) investigated mass and heat transport and investigated the effects of parameters, the enhanced thermal radiation diminishes temperature and velocity, the improved chemical reaction drops concentration, and the increased Soret parameter increases both temperature and velocity, according to numerical results. TiO_2 nanofluids-based water flows more quickly than Cu nanofluids. Khan et al. (2023) examine the effects of thermal radiation on vertical Riga plate. The study investigates numerical solutions, providing specific impacts on skin friction, temperature, velocity, concentration and *Nu*.

4. Justification for selection of research area

The selection of a research area/topic for a nanofluid model for heat and mass transfer performance is likely to be driven by a variety of factors, including

(1) **Scientific and technological relevance:** Science and technology must examine the thermal properties and fluid flow of nanofluids when nanoparticles are present, heat transfer efficiency improves drastically. It can provide insights into the behaviour of these fluids in various types of industries mostly used in heat exchanger and biomedical applications.

(2) **Novelty and originality:** The study of various types of nanofluids and check the dependency of thermophysical characteristics for better heat and mass transfer performance. Previous investigations and available data help us to develop a new model to study this area can be considered an innovative and original research area.

(3) **Practical applications:** Understanding the flow behavior of nanofluids on various types of geometries can have practical applications in a wide range of industries, including energy, biomedical engineering, heat exchangers, and many others.

5. Research objectives

(1) To study the various models for heat and mass transfer of nanofluids on various geometries.

(2) To understand the influence of nanosized particles on flow behavior across the chosen geometrical structure.

(3) By studying previous models, develop a new mathematical model to study the behavior of the nanofluid over a chosen geometry.

(4) To assess the performance of heat and mass transfer by examining the thermophysical characteristics of nanofluids influenced by various factors.

6. Research Methodology

Here we go through with general methodology as it varies with the research objective, methodology could include some steps:

1) Literature review: We will try to comprehend the current state of research in nanofluids and the impact of geometry on its flow, by available resources and literature articles.
2) Developing the mathematical model: We will create a mathematical model based on thermodynamic and fluid dynamic equations, which includes the changes due to the presence of nanoparticles and the impact of geometry on flow.
3) Thermophysical characteristics: We will measure the thermophysical characteristics of the nanofluid, including viscosity, thermal conductivity, and density, using appropriate techniques.
4) Solving the equations: We will employ both analytical (Laplace transformation) and numerical methods, including but not limited to the finite difference method, shooting technique, and Matlab boundary value problem solver bvp4c, as needed, to address the formulated problems. The selection of the appropriate solution method will be based on the governing differential equations.
5) Plotting the graphs: After applying numerical techniques to the nanofluid flow across the chosen shape, we will produce the graphs for the simplified differential equations, which include multiple parameters that measure variables like temperature, velocity, and nanoparticle concentration.

6) Analyzing and interpreting the results: We will analyze and interpret the results of the model chosen, check the previous research data, and conclude the behavior of the nanofluid.
7) Write a research paper: After all, when we complete our research work and reach some conclusion we will write a research paper detailing the work, including a description of the methodology, results, and conclusions.

7. Expected Outcome

The expected outcome or hypothesis for a nanofluid model using thermophysical conditions over the geometry will depend on the specific research objectives and the variables being studied. However, some general outcomes or hypotheses that researchers might expect or investigate include: The thermophysical characteristics of the nanofluid will be influenced by the nanoparticle's presence, and these changes will be reflected in the flow behavior of the nanofluid over the chosen geometry.

(1) Due to the nanoparticle's occurrence in fluid, flow behavior at geometrical structure changes, and also it leads to changes in the velocity, temperature profile, and concentration profile.
(2) To validate the model, we will check whether the prediction by the proposed model matches the already available model. If, it matches that indicates our model can be used for the chosen geometry to create a prediction.

8. Work Plan

Research Activity	6	12	18	24	30	36	*
1.CourseWork							
2.LiteratureSurvey							
3.MethodologySteps							
4.QuaterlyReports							
5.CommunicationofPapers							
6.PreparationofFinalThesis							
7.SubmissionofThesis							

*As approved Extension

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